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CHORDS HALVING THE AREA OF A PLANAR CONVEX SET

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Abstract. Let $K \subset \mathbb{R}^2$ be a compact convex set in the plane. A halving chord of K is a line segment $p\hat{p}$, $p, \hat{p} \in \partial K$, which divides the area of K into two equal parts. For every direction v there exists exactly one halving chord. Its length $h_A(v)$ is the corresponding (area) halving distance. In this article we give inequalities relating the minimum and maximum (area) halving distance h_A and H_A of a convex closed region $K \subset \mathbb{R}^2$ to other geometric quantities of K, namely the minimal width ω, the diameter D, the perimeter p, the inradius r, the circumradius R, and the area A. We try to find tight inequalities, and characterize their extremal sets (the sets attaining equality).

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