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THE STABILITY PROBLEM OF THE HERMITE–HADAMARD INEQUALITY

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Abstract. The problem of the Hyers-Ulam stability of the Hermite-Hadamard inequality posed by Zs. Páles is solved. It is shown that for continuous functions $f: I \to \mathbb{R}$ neither the inequality $f\left(\frac{x+y}{2}\right) \leqslant \frac{1}{y-x}\int_x^y f(t)\,dt + \epsilon$ nor $\frac{1}{y-x}\int_x^y f(t)\,dt \leqslant \frac{f(x)+f(y)}{2} + \epsilon$ implies the $c\epsilon-$ convexity of f (with any $c > 0$). However, if f is continuous and satisfies both of the above inequalities simultaneously, then it is 4ϵ-convex.

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