

## A NEW CLASS OF GENERALIZED NONLINEAR MULTI-VALUED QUASI-VARIATIONAL-LIKE INCLUSIONS WITH $H$ -MONOTONE MAPPINGS

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**Abstract.** In this paper, we introduce and study a new class of generalized nonlinear multi-valued quasi-variational-like inclusions with  $H$ -monotone operators in Hilbert spaces. By using the resolvent operator method associated with  $H$ -monotone operator due to Fang and Huang, we construct a new iterative algorithm for solving this kind of nonlinear multi-valued variational inclusions. We also prove the existence of solutions for the nonlinear multi-valued variational inclusions and the convergence of iterative sequences generated by the algorithm. Our results improve and generalize many known corresponding results.

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*Key words and phrases:* generalized nonlinear multi-valued variational inclusion with  $H$ -monotone operator, resolvent operator technique, relaxed cocoercive mapping, iterative algorithm with errors, existence and convergence.

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