SHARP TRIANGLE INEQUALITY AND
ITS REVERSE IN BANACH SPACES

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Abstract. We shall present a sharp triangle inequality and its reverse inequality with n elements in a Banach space X, or equivalently we shall estimate the difference $\sum_{j=1}^{n} \|x_j\| - \|\sum_{j=1}^{n} x_j\|$ for given $x_1, x_2, \ldots, x_n$ in X, where equality attainedness will be characterized. Several applications will be given.

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