

ANTI EIGENVALUES AND ANTISINGULARVALUES OF A MATRIX AND APPLICATIONS TO PROBLEMS IN STATISTICS

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Abstract. Different ways of characterizing antieigen and antisingularvalues are considered. Several matrix inequalities, their generalizations and applications to problems in multivariate statistics are given. A new concept of homologous canonical correlations is introduced and applied to a problem in genetics.

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