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CONTRACTED MODULAR DIOPHANTINE INEQUALITIES AND NUMERICAL SEMIGROUPS

J. C. ROSALES AND J. M. URBANO-BLANCO

Abstract. We study the set $\mathrm{T}(a,b,c)$ of all integer solutions to the Diophantine inequality $ax \bmod b \leqslant x - c$, with a,b,c nonnegative integers and $b \neq 0$. We obtain an exact formula for the cardinality of $\mathbb{N} \setminus \mathrm{T}(a,b,c)$ and give an algorithm to decide whether or not a numerical semigroup can be represented as $\mathrm{T}(a,b,c) \cup \{0\}$.

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