

SOME NEW INEQUALITIES BETWEEN IMPORTANT MEANS AND APPLICATIONS TO KY FAN – TYPE INEQUALITIES

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Abstract. In this paper, mainly using the convexity of the function $\frac{a^x - b^x}{c^x - d^x}$ and convexity or concavity of the function $\ln \frac{a^x - b^x}{c^x - d^x}$ on the real line, where $a > b \geq c > d > 0$ are fixed real numbers, we obtain some important relations between various important means of these numbers. Also, we apply the obtained results to Ky Fan type inequalities and get some new refinements.

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