

## A FURTHER GENERALIZATION OF ACZÉL'S INEQUALITY AND POPOVICIU'S INEQUALITY

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**Abstract.** In this paper, a new generalization of Aczél's inequality is established, which contains as special case a sharpened version of Popoviciu's inequality:

$$\left( a_1^p - \sum_{i=2}^n a_i^p \right)^{\frac{1}{p}} \left( b_1^q - \sum_{i=2}^n b_i^q \right)^{\frac{1}{q}} \leq a_1 b_1 - \left( \sum_{i=2}^n a_i b_i \right) - \frac{a_1 b_1}{\max\{p, q\}} \left( \sum_{i=2}^n \left( \frac{a_i^p}{a_1^p} - \frac{b_i^q}{b_1^q} \right) \right)^2,$$

where  $p, q, a_i, b_i$  ( $i = 1, 2, \dots, n$ ) are positive numbers,  $p^{-1} + q^{-1} = 1$ ,  $a_1^p - \sum_{i=2}^n a_i^p > 0$  and  $b_1^q - \sum_{i=2}^n b_i^q > 0$ . Moreover, an integral inequality of Aczél-Popoviciu type is given.

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