

ASYMPTOTIC BEHAVIOR OF NONOSCILLATORY SOLUTIONS OF SECOND ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. We study asymptotic behavior of solutions of second order nonlinear neutral differential equations of the form

$$(x(t) + p(t)x(t - \tau))'' + f(t, x(t), x(\rho(t)), x'(t), x'(\sigma(t))) = 0.$$

First we prove that solutions can be indefinitely continued to the right. Then, using the celebrated Bihari integral inequality, we obtain conditions for all nonoscillatory solutions to behave at infinity like nontrivial linear functions. Our theorems complement and extend recent results reported in the literature.

Mathematics subject classification (2000): 34K40, 26D10, 34D05, 34K25.

Key words and phrases: nonlinear neutral differential equations, second order, asymptotic behavior, nonoscillatory solutions, indefinite continuation, Bihari inequality.

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