TWO EXTRAPOLATION THEOREMS FOR
RELATED WEIGHTS AND APPLICATIONS

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Abstract. In this paper we prove two extrapolation theorems for related weights. The theorems proved by C. Segovia and J. L. Torrea in [C. Segovia and J. L. Torrea, *Weighted inequalities for commutators of fractional and singular integrals*, Publ. Mat. **35**, (1991), 209–235] are adapted for one-sided weights. We apply these extrapolation theorems to improve weighted inequalities for commutators (with symbol b depending on the related weights) of several one-sided operators such as the Weyl and the Riemann-Liouville fractional integrals, or one-sided maximal operators given by the convolution with a smooth function. We also characterize the symbols b for which the commutators of these one-sided operators are bounded.

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ⓒ ℰℒℰℳℰℕ, Zagreb
Paper MIA-10-60

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