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BESSEL POTENTIAL SPACES WITH VARIABLE EXPONENT

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Abstract. We show that a variable exponent Bessel potential space coincides with the variable exponent Sobolev space if the Hardy-Littlewood maximal operator is bounded on the underlying variable exponent Lebesgue space. Moreover, we study the Hölder type quasi-continuity of Bessel potentials of the first order.

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REFERENCES

[1] A. ALMEIDA, S. SAMKO, *Characterization of Riesz and Bessel potentials of variable Lebesgue spaces*, J. Funct. Spaces Appl., **4**, (2) (2006), 113–144.
[2] D. CRUZ-URIBE, A. FIORENZA AND C. J. NEUGEBAUER, *The maximal operator on variable L^p spaces*, Ann. Acad. Sci. Fenn., **28**, (2003), 223–238.
[3] D. CRUZ-URIBE, A. FIORENZA AND C. J. NEUGEBAUER, *Corrections to "The maximal operator on variable L^p spaces"*, Ann. Acad. Sci. Fenn., **29**, (2004), 247–249.
[4] D. CRUZ-URIBE, A. FIORENZA, J. M. MARTELL AND C. PÉREZ, *The boundedness of classical operators on variable L^p spaces*, Ann. Acad. Sci. Fenn., **31**, (2006), 239–264.
[5] L. DIENING, *Maximal function on generalized Lebesgue spaces $L^{p(\cdot)}$*, Math. Inequal. Appl., **7**, (2) (2004), 245–253.
[6] L. DIENING, *Maximal function on Orlicz-Musiełak spaces and generalized Lebesgue spaces*, Bull. Sci. Math., **129**, (8) (2005), 657–700.
[7] L. DIENING, *Riesz potential and Sobolev embeddings on generalized Lebesgue and Sobolev spaces $L^{p(\cdot)}$ and $W^{k,p(\cdot)}$*, Math. Nachr., **263**, (1) (2004), 31–43.
[8] L. DIENING, M. RŮŽIČKA, *Calderón-Zygmund operators on generalized Lebesgue spaces $L^{p(\cdot)}$ and problems related to fluid dynamics*, J. Reine Angew. Math., **563**, (2003), 197–220.
[9] D. E. EDMUNDS, J. RÁKOSNÍK, it Sobolev embeddings with variable exponent, Studia Math., **143**, (3) (2000), 267–293.
[10] L. C. EVANS, R. F. GARIEPY, *Measure Theory and Fine Properties of Functions*, CRC Press, Boca Raton, 1992.
[11] P. HARJULEHTO, P. HÄSTÖ, *Lebesgue points in variable exponent spaces*, Ann. Acad. Sci. Fenn. Math., **29**, (2004), 295–306.
[12] P. HARJULEHTO, P. HÄSTÖ, M. KOSKENOJA AND S. VARONEN, it Sobolev capacity on the space $W^{1,p(\cdot)}(\mathbb{R}^n)$, J. Funct. Spaces Appl., **1**, (1) (2003), 17–33.
[13] J. KINNUNEN, O. MARTIO, *The Sobolev capacity on metric spaces*, Ann. Acad. Sci. Fenn. Math., **21**, (1996), 367–382.
[14] O. KOVÁČIK, J. RÁKOSNÍK, *On spaces $L^{p(x)}$ and $W^{1,p(x)}$*, Czechoslovak Math. J., **41**, (116) (1991), 592–618.
[15] A. K. LERNER, *Some remarks on the Hardy-Littlewood maximal function on variable L^p spaces*, Math. Z., **251**, (3) (2005), 509–521.
[16] J. MALÝ, *Hölder type quasicontinuity*, Potential Anal., **2**, (3) (1993), 249–254.

[17] A. NEKVINDA, *A note on maximal operator on* $\ell^{\{p_n\}}$ *and* $L^{p(x)}(\mathbb{R})$, J. Funct. Spaces Appl., **5**, (1) (2007), 49–88.

[18] A. NEKVINDA, *Hardy–Littlewood maximal operator on* $L^{p(x)}(\mathbb{R}^n)$, Math. Inequal. Appl., **7**, (2) (2004), 255–265.

[19] E. NIEMINEN, *Hausdorff measures, capacities, and Sobolev spaces with weights*, Ann. Acad. Sci. Fenn. Ser. A I Math. Dissertationes, **81**, (1991), 39 pp.

[20] L. PICK, M. RŮŽIČKA, *An example of a space* $L^{p(x)}$ *on which the Hardy–Littlewood maximal operator is not bounded*, Expo. Math., **19**, (2001), 369–371.

[21] M. RŮŽIČKA, *Electrorheological Fluids: Modeling and Mathematical Theory*, Springer-Verlag, Berlin, 2000.

[22] S. SAMKO, *Denseness of* $C_0^\infty(\mathbb{R}^n)$ *in the generalized Sobolev spaces* $W^{m,p(x)}(\mathbb{R}^n)$, pp. 333–342 in Direct and inverse problems of mathematical physics (Newark, DE, 1997), Int. Soc. Anal. Appl. Comput., bf 5, Kluwer Acad. Publ., Dordrecht, 2000.

[23] E. STEIN, *Singular integrals and differentiability properties of functions*, Princeton University Press, Princeton, 1970.

[24] H. TRIEBEL, *Theory of function spaces*, Basel, Birkhäuser, 1983.