SEVERAL INEQUALITIES FOR THE LARGEST SINGULAR VALUE AND THE SPECTRAL RADIUS OF MATRICES

SHU-QIAN SHEN AND TING-ZHU HUANG

Abstract. For nonnegative matrices $A = (a_{ij}) \in \mathbb{R}^{n \times m}$, $B = (b_{ij}) \in \mathbb{R}^{m \times n}$ and any $t \in [0, 1]$, we present $\sigma(S_t(A, B)) \leqslant \sigma(A)^t \sigma(B)^{1-t}$, in which $S_t(A, B) = (a_{ij}^t b_{ji}^{1-t})$ and $\sigma(\cdot)$ denotes the largest singular value. Using the result obtained, the inequality $\sigma(A \circ B) \leqslant \sqrt{\sigma(A \circ A)\sigma(B \circ \overline{B})}$ for matrices $A = (a_{ij})$ and $B = (b_{ij}) \in \mathbb{C}^{n \times m}$ is established. Here, $A \circ B = (a_{ij}\overline{b_{ij}})$, and $\overline{b_{ij}}$ denotes the complex conjugate of b_{ij}. Finally, some inequalities for the spectral radius are also studied.

REFERENCES

[1] A. J. SCHWENK, *Tight bounds on the spectral radius of asymmetric nonnegative matrices*, Linear Algebra Appl., **75** (1986), 257–265
[2] R. S. VARGA, *Matrix Iterative Analysis* (Second Edition), Springer-Verlag, Berlin Herdelberg, 2000.
[3] L. YU. KOLOTILINA, *Lower bounds for the Perron root of a nonnegative matrix*, Linear Algebra Appl., **180** (1993), 133–151.
[4] O. ROJO, R. SOTO, H. ROJO, *Bounds for the spectral radius and the largest singular value*, Comput. Math. Appl., **36** (1998), 41–50.
[5] D. S. MITRINOVIC, *Analytic Inequalities*, Springer-Verlag, New York , 1970.
[6] L. ELSNER, D. HERSHKOWITZ, A. PINKUS, *Functional inequalities for spectral radii of nonnegative matrices*, Linear Algebra Appl., **129** (1990), 103–130.
[7] S. KARLIN, F. OST, *Some monotonicity properties of Schur powers of matrices and related inequalities*, Linear Algebra Appl., **68** (1985) 47–65.
[8] R. A. HORN, C. A. JOHNSON, *Matrix Analysis*, Cambridge University Press, Cambridge, 1985.
[9] H. MINC, *Nonnegative Matrices*, Academic Press, New York, 1988.
[10] A. BERMAN, R. PLEMMONS, *Nonnegative Matrices in the Mathematical Sciences*, Academic Press, New York, San Francisco, London, 1979.

ⓒ ELEM, Zagreb
Paper MIA-10-65

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com