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REGULARITY RESULTS FOR DEGENERATE ELLIPTIC EQUATIONS RELATED TO GAUSS MEASURE

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Abstract. In this paper we study a Dirichlet problem relative to the equation $Lu = g\varphi - (f_i\varphi)_{x_i}$, where L is a linear elliptic operator with lower-order terms whose ellipticity condition is given in terms of the function $\varphi(x) = (2\pi)^{-\frac{n}{2}} \exp\left(-|x|^2/2\right)$, the density of the Gaussian measure. We use the notion of rearrangement with respect to the Gauss measure to obtain a priory estimate of the solution u and we study the summability of u in the Lorentz-Zygmund spaces when g and f_i varies in suitable Lorent-Zygmund spaces.

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