

## INTERNAL CUBIC SYMMETRIC FORMS IN A SMALL NUMBER OF VARIABLES

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*Abstract.* We consider means of the form  $\mu = \sqrt[3]{f}$ , where  $f$  is a cubic symmetric form in  $n$  variables, and we show that if  $n \leq 4$  and  $\mu$  is internal on the points  $(x_1, \dots, x_n)$  where  $x_i = 0$  or 1, then  $\mu$  is internal for all points  $(x_1, \dots, x_n)$  with  $x_i \geq 0$  for all  $i$ . We highlight the similarity between this internality problem and the parallel problem pertaining to copositive symmetric cubic forms.

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