

ON THE HYERS–ULAM–RASSIAS STABILITY PROBLEM FOR APPROXIMATELY k -ADDITIVE MAPPINGS AND FUNCTIONAL INEQUALITIES

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Abstract. The purpose of this paper is to solve the generalized Hyers–Ulam stability problem for a k -additive functional equation

$$D_{m,i}f(x, y) = 0,$$

on the basis of direct method, where $k=4m+i$ is a positive integer for each $i = -1, 0, 1$ and 2 .

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