# ON $n$–TH JAMES AND KHINTCHINE CONSTANTS
# OF BANACH SPACES

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*Abstract.* For any Banach space $X$ the $n$-th James constants $J_n(X)$ and the $n$-th Khintchine constants $K_{p,q}^n(X)$ are investigated and discussed. Some new properties of these constants are presented. The main result is an estimate of the $n$-th Khintchine constants $K_{p,q}^n(X)$ by the $n$-th James constants $J_n(X)$. In the case of $n=2$ and $p=q=2$ this estimate is even stronger and improvs an earlier estimate proved by Kato-Maligranda-Takahashi [25].

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