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THE REVERSE HARDY INEQUALITY WITH MEASURES

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Abstract. In this paper we characterize the validity of the inequalities

$$\|g\|_{p,(a,b),\lambda} \leqslant c \left\| u(x) \int_{(a,x)} g(y)\, d\mu \right\|_{q,(a,b),\nu}$$

and

$$\|g\|_{p,(a,b),\lambda} \leqslant c \left\| u(x) \int_{(x,b)} g(y)\, d\mu \right\|_{q,(a,b),\nu}$$

for non-negative Borel measurable functions g on the interval $(a,b) \subseteq \mathbb{R}$, where $0 < p \leqslant 1$, $0 < q \leqslant +\infty$, λ, μ and ν are non-negative Borel measures on (a,b), and u is a weight function on (a,b).

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