# INEQUALITY OF O'NEIL–TYPE FOR CONVOLUTIONS ASSOCIATED WITH THE LAPLACE–BESSEL DIFFERENTIAL OPERATOR AND APPLICATIONS

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*Abstract.* In this paper we prove an O'Neil-type inequality for the convolution operator ($B$–convolution) associated with the Laplace-Bessel differential operator. By using an O'Neil-type inequality for rearrangements we obtain a pointwise rearrangement estimate of the $B$–convolution. As an application, we obtain necessary and sufficient conditions on the parameters for the boundedness of the fractional $B$-maximal operator and $B$–fractional integral operator with rough kernels from the spaces $L_{p,\gamma}$ to $L_{q,\gamma}$ and from the spaces $L_{1,\gamma}$ to the weak spaces $WL_{q,\gamma}$.

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