

GENERALIZED QUASILINEARIZATION METHOD FOR A FORCED DUFFING EQUATION WITH THREE-POINT NONLINEAR BOUNDARY CONDITIONS

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Abstract. We develop a generalized quasilinearization method for a forced Duffing equation with three-point nonlinear boundary conditions and obtain two monotone sequences of approximate solutions converging quadratically to the unique solution of the problem.

Mathematics subject classification (2000): 34A37, 34B15.

Key words and phrases: Quasilinearization, Duffing equation, quadratic convergence.

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