

THE SMALLEST POSITIVE INTEGER THAT IS SOLUTION OF A PROPORTIONALLY MODULAR DIOPHANTINE INEQUALITY

J. C. ROSALES AND P. VASCO

Abstract. A proportionally modular Diophantine inequality is an expression of the form $ax \bmod b \leq cx$, where a , b and c are positive integers. In this paper we present an algorithm that allows us to calculate the smallest positive integer that is solution of an inequality of this type. We also obtain an algorithm that computes the Frobenius number and the number of gaps of a numerical semigroup generated by three positive integers.

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