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GENERAL HILBERT–TYPE INEQUALITIES WITH NON–CONJUGATE EXPONENTS

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Abstract. In this paper we derive a series of new one-dimensional and multidimensional integral and discrete inequalities of the Hilbert and the Hardy-Hilbert type, with non-conjugate exponents. First, prove and discuss two equivalent general inequalities of such type, as well as their corresponding reverse inequalities. The obtained results are then applied to various settings considering homogeneous functions of a negative real degree. In particular, we prove generalizations and refinements of some recent results of Rassias et al, related to the Hilbert-type inequalities with conjugate exponents, and some new multidimensional inequalities of the Godunova type.

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