Mathematical
Inequalities
& Applications
Volume 11, Number 2 (2008), 283–296

L_q NORM INEQUALITIES FOR THE POLAR DERIVATIVE OF A POLYNOMIAL

A. AZIZ, N. A. RATHER AND Q. ALIYA

Abstract. If $P(z)$ is a polynomial of degree n which does not vanish in $|z| < 1$, then it was shown by Govil, Nyuydinkong and Tameru [Some L_p inequalities for the polar derivative of a polynomial, J. Math. Anal. Appl., **254**, (2001), 618–626] that for every real or complex number α with $|\alpha| \leqslant 1$ and $q \geqslant 1$,

$$|D_\alpha P| \leqslant \left(\frac{|\alpha|+1}{\|1+z\|_q}\right)\|P\|_q$$

where $D_\alpha P(z)$ denotes the polar derivative of $P(z)$ with respect to $\alpha \in C$. Unfortnunately the proof of this result is not correct. In this paper, we prove a more general result which not only provides a correct proof of this result but also extends some known L_q norm inequalities for the polar derivative of a polynomial. We also present L_q norm inequality for polynomials not vanishing in $|z| > k$ where k $\leqslant 1$.

Mathematics subject classification (2000): 26D05, 30D15, 41A17.
Key words and phrases: Zygmund's inequality, polar derivative, L_q -norm.

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