

GENERALIZED HYERS—ULAM STABILITY OF MAPPINGS ON NORMED LIE TRIPLE SYSTEMS

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Abstract. We prove the generalized Hyers–Ulam stability of mappings on normed spaces for the Pexiderized Cauchy–Jensen additive mapping

$$f\left(\frac{x+y}{2} + z\right) + g\left(\frac{x-y}{2} - z\right) = h(x).$$

Then we apply the results for investigating the stability of homomorphisms and derivations on normed Lie triple systems.

Mathematics subject classification (2000): 39B82, 16W25, 17A40, 39B52, 47Jxx.

Key words and phrases: Pexiderized Cauchy–Jensen additive mapping; normed Lie system; homomorphism; derivation; generalized Hyers–Ulam stability.

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