Mathematical
Inequalities
& Applications
Volume 11, Number 3 (2008), 393–413

HARDY INEQUALITY WITH THREE MEASURES ON MONOTONE FUNCTIONS

MARIA JOHANSSON, VLADIMIR D. STEPANOV AND ELENA P. USHAKOVA

Abstract. Characterization of $L^p_v[0,\infty) - L^q_\mu[0,\infty)$ boundedness of the general Hardy operator $(H_s f)(x) = \left(\int_{[0,x]} f^s u d\lambda\right)^{\frac{1}{s}}$ restricted to monotone functions $f \geqslant 0$ for $0 < p,q,s < \infty$ with positive Borel σ -finite measures λ, μ and v is obtained.

Mathematics subject classification (2000): 26D10, 26D15, 26D07.
Key words and phrases: Integral inequalities, weights, Hardy operator, monotone functions, measures.

REFERENCES

[1] BENNETT G. AND GROSSE-ERDMANN K.-G., *Weighted Hardy inequality for decreasing sequences and functions.* Math. Ann., 334 (2006), 489–531.
[2] GOLDMAN, M. L., *Sharp estimates for the norms of Hardy-type operators on cones of quasimonotone functions.* Proc. Steklov Inst. Math. 2001, no. 1 (232), 109–137.
[3] KUFNER A., MALIGRANDA L. AND PERSSON L.-E., *The Hardy inequality - About its history and some related results.* Publishing House, Pilsen, 2007.
[4] KUFNER A. AND PERSSON L.-E., *Weighted inequalities of Hardy type*, World Scientific, Singapore/New Jersey/ London/Hong Kong, 2003.
[5] PERSSON L.-E., STEPANOV V. D. AND USHAKOVA E. P., *Equivalence of Hardy-type inequalities with general measures on the cones of non-negative respective non-increasing functions.* Proc. Amer. Math. Soc., (8) 134 (2006), 2363–2372.
[6] PROKHOROV D. V., *Hardy's inequality with three measures.* Proc. Steklov Inst. Math., 255 (2007), 233-242.
[7] PROKHOROV D. V., *Inequalities of Hardy type for a class of integral operators with measures.* Anal. Math 33 (2007), 199–225.
[8] ROYDEN H. L., *Real analysis.* Third edition. Macmillan Publishing Company, New York, 1988.
[9] SAWYER E., *Boundedness of classical operators on classical Lorentz spaces.* Studia Math., 96 (1990), 145–158.
[10] SINNAMON G., *Transferring monotonicity in weighted norm inequalities.* Collect. Math., 54 (2003), 181-216.
[11] SINNAMON G., *Hardy's inequality and monotonocity. In: Function Spaces and Nonlinear Analysis* (Eds.: P. Drábec and J. Rákosnik), Mathematical Institute of the Academy of Sciences of the Czech Republic, Prague, 2005, 292–310.
[12] SINNAMON, G., STEPANOV, V. D., *The weighted Hardy inequality: new proofs and the case p=1.* J. London Math. Soc. (2) 54 (1996), no. 1, 89–101.
[13] STEPANOV V. D., *The weighted Hardy's inequality for nonincreasing functions.* Trans. Amer. Math. Soc., (1) 338 (1993), 173–186.

Paper MIA-11-30