

## ON CERTAIN MEAN VALUE THEOREMS

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*Abstract.* In this paper we have obtained a symmetric integral mean  $M(a, b; p(r_{n,k}), q)$  involving functions which a generalization of the arithmetic-geometric mean of Gauss. We have also proved some characterization of the symmetric mean values for the twice continuously differentiable function  $p$ .

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*Key words and phrases:* Arithmetic-geometric mean of Gauss, symmetric mean, monotonic function, partial derivative, differential equation.

### REFERENCES

- [1] S. ABRAMOVICH AND J. PEČARIĆ, *New properties of some mean values*, J. Math. Anal. Appl. **232** (1999), 376-383.
- [2] H. ALZER, *On Stolarsky's mean value family*, Int. J. Math. Educat. Sci. Tech. **20** (1) (1987), 186-189.
- [3] P.S. BULLEN, D.S. MITRINOVIĆ AND P.M. VASIĆ, *Mean and Their Inequalities*, Reidel, Dordrecht, 1988.
- [4] P.S. CARLSON, *The logarithmic mean*, Amer. Math. Monthly **79** (1972), 615-618.
- [5] L. GALVANI, *Dei limiti a sui tendono alcune media*, Bol. Un. Mat. Ital. **6** (1927), 173-179.
- [6] D.S. MITRINOVIĆ, J.E. PEČARIĆ AND A.M. FINK, *Classical and new Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht, Boston, London, 1993.
- [7] H. HARUKI, *New characterizations of the arithmetic-geometric mean of Gauss and other well-known values*, Publ. Math. Debrecen **38** (1991), 323-332.
- [8] H. HARUKI AND T.M. RASSIAS, *New characterizations of some mean-values*, Jour. of Mathematical Analysis and Applications **202** (1996), 333-348.
- [9] H. HARUKI AND TH.M. RASSIAS, *A new analogue of Gauss' functional equation*, Internat. Journal of Math. and Math. Sciences **18** (1995), 749-756.
- [10] P. KAHLIG AND J. MATKOWSKI, *On the composition of homogeneous quasi-arithmetic means*, Jour. of Mathematical Analysis and Applications **216** (1997), 69-85.
- [11] Y.H. KIM, *New characterizations of well-known mean-values*, Far East J. Math. Sci. **6**(6) (1998), 939-947.
- [12] Y.H. KIM AND TH.M. RASSIAS, *Properties of some mean values and functional equations*, Panamer. Math. J. **12**(1) (2002), 65-74.
- [13] G. PÓLYA AND G. SZEGÖ, *Isoperimetric Inequalities in Mathematical Physics*, Princeton University Press, 1951.
- [14] F. QI, *Generalized weighted mean values with two parameters*, Proc. Royal Soc. London A. **454** (1998), 2723-2732.
- [15] P.K. SAHOO AND T. RIEDEL, *Mean Value Theorems and Functional Equations*, World Scientific Publ. Co., Singapore, New Jersey, London, 1998.
- [16] K.B. STOLARSKY, *Generalizations of the logarithmic mean*, Math. Mag. **48** (1975), 87-92.
- [17] GH. TOADER, *Some mean values related to the arithmetic-geometric mean*, Jour. of Mathematical Analysis and Applications **218** (1998), 358-368.
- [18] GH. TOADER AND T.M. RASSIAS, *New properties of some mean values*, Jour. of Mathematical Analysis and Applications **232** (1999), 376-383.

- [19] S. TOADER, TH.M. RASSIAS AND G. TOADER, *A Gauss type functional equation*, Int. J. Math. Math. Sci. **25(9)** (2001), 565-569.