# MAJORIZATION INEQUALITIES RELATED TO INCREASING CONVEX FUNCTIONS IN A SEMIFINITE VON NEUMANN ALGEBRA

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*Abstract.* Let $\mu_s(x)$ denote the generalized $s$-number of an operator $x$. We show a majorization inequality $\int_0^t \mu_s(f(a+b))\,ds \geqslant \int_0^t \mu_s(f(a)+f(b))\,ds$ for every increasing convex function with $f(0) = 0$ and positive $\tau$-measurable operators $a$, $b$ affiliated with a semi-finite von Neumann algebra $\mathfrak{A}$.

## REFERENCES

[1] T. ANDO, X. ZHAN, *Norm inequalities related to operator monotone functions*, Math. Ann., **315** (1999), 771–780.
[2] R. BHATIA, *Matrix Analysis*, Springer, New York, 1996.
[3] J.C. BOURIN, *Some inequalities for norms on matrices and operators*, Linear Algebra Appl., **292** (1999), 139–154.
[4] T. FACK AND H. KOSAKI, *Generalized s-numbers of $\tau$-measurable operators*, Pacific J. Math., **23** (1986), 269–300.
[5] T. KOSEM, *Inequalities between $||f(A+B)||$ and $||f(A)+f(B)||$*, Linear Algebra Appl., **418** (2006), 153-160.
[6] F. HIAI AND Y. NAKAMURA, *Majorizations for generalized s-numbers in semifinite von Neumann algebras*, Math. Z., **195** (1987), 17–27.
[7] E. NELSON, *Notes on non-commutative integration*, J. Funct. Anal., **15** (1974), 103–116.
[8] I. SEGAL, *A non-commutative extension of abstract integration*, Ann. of Math., **57** (1953), 401–457.
[9] E. STEIN AND G. WEISS, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton Univ. Press, 1971.

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