A SHARPENED VERSION OF THE
FUNDAMENTAL TRIANGLE INEQUALITY

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Abstract. In this note, we show a sharpened version of the classical fundamental triangle inequality, as follows

$$2R^2+10Rr-r^2-2(R-2r)\sqrt{R^2-2Rr}\cos\phi \leqslant s^2 \leqslant 2R^2+10Rr-r^2+2(R-2r)\sqrt{R^2-2Rr}\cos\phi,$$

where $\phi = \min_{1\leqslant i<j\leqslant 3}\left|A_i - A_j\right|$.

REFERENCES

[1] É. ROUCHÉ AND RAMUS, *Question 233*, Nouv. Ann. Math., **10**(1851), 353–355.
[2] D. S. MITRINOVIĆ, J. E. PEČ ARIĆ AND V. VOLENEC, *Recent Advances in Geometric Inequalities*. Kluwer Academic Publishers, Dordrecht, Netherlands, 1989.
[3] O. BOTTEMA, R. Z. DJORDJEVIĆ, R. R. JANIĆ, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Geometric Inequalities*. Wolters-Noordhoff, Groningen, 1969.
[4] D. S. MITRINOVIĆ, J. E. PEČ ARIĆ, V. VOLENEC AND J. CHEN, *Addenda to the Monograph: Recent Advances in Geometric Inequalities(I)*. Journal of Ningbo University, **4**(2)(1991)80–92.
[5] Z. SHAN, *Geometric Inequality in China*. Jiangsu Education Publishing House, Nanjing, 1996 (in Chinese).
[6] J. C. KUANG, *Applied Inequalities*, 2nd ed., Shandong Science and Technology Press, Jinan, 1993 (in Chinese).
[7] R. A. SATNOIANU, *General power inequalities between the sides and the circumscribed and inscribed radii related to the fundamental triangle inequality*. Math. Inequal. Appl., **5**(4)(2002), 745–751.
[8] R. A. SATNOIANU, *Refined geometric inequalities between two or more triangles obtained by dedublation*. Math. Inequal. Appl., **7**(2)(2004), 289–298.
[9] S. WU, *Generalization and sharpness of Finsler-Hadwiger's inequality and its applications*. Math. Inequal. Appl., **9** (3) (2006), 421–426.
[10] SH-H. WU AND ZH-H. ZHANG, *A class of inequalities related to the angle bisectors and the sides of a triangle*. J. Inequal. Pure Appl. Math., **7** (3) (2006), Article 108.