GENERALIZED RIESZ PROJECTIONS AND
TOEPLITZ OPERATORS

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Abstract. Let $1 < p < \infty$. In this paper, for a measurable function v and a weight function w, the generalized Riesz projection P^v is defined by $P^v f = vP(v^{-1}f)$, $(f \in L^p(w))$. If P_0 is the self-adjoint projection from $L^2(w)$ onto $H^2(w)$, then $P_0 = P^\alpha$ for some outer function α satisfying $w = |\alpha|^{-2}$. In this paper, P^v on $L^p(w)$ is studied. As an application, the invertibility criterion for the generalized Toeplitz operator T^v_ϕ and the generalized singular integral operator $\phi P^v + Q^v$, $Q^v = I - P^v$ are investigated using the weighted norm inequality. The operator norm inequality for the generalized Hankel operator H^v_ϕ is also presented.

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ⓒ ELEMENT, Zagreb
Paper MIA-11-40

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