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A CONDITION OF UNIFORM EXPONENTIAL STABILITY FOR SEMIGROUPS

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Abstract. The aim of this paper is to prove that the uniform exponential stability of a strongly continuous semigroup $\{T(t)\}_{t\geqslant 0}$ (acting on a complex Hilbert space H) can be derived as a consequence of the well behavior of its numerical range in a suitable Orlicz space. More precisely, assuming that there exists an Orlicz space $E=(L^{\Phi},\rho^{\Phi})$ over $\mathbf{R}_+$ such that

$$\liminf_{\alpha\downarrow 0}[\alpha||\exp_{-\alpha}||_{E^{\star}}]=0$$

and

$$\sup_{||x||\leqslant 1}\rho^{\Phi}(|\langle T(\cdot)x,x\rangle|)\leqslant M<\infty,$$

then the uniform growth bound ω_0 of the semigroup verifies an estimate of the form

$$\omega_0\leqslant M_{\beta}:=\beta-(2M||\exp_{-\beta}||_{E^{\star}})^{-1}<0$$

for some positive number β. As an application, the well posedness of an abstract infinite time Cauchy problem is discussed.

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