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THE APPROXIMATION OF POWER FUNCTION BY THE q–BERNSTEIN POLYNOMIALS IN THE CASE $q > 1$

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Abstract. Since for $q > 1$, q-Bernstein polynomials are not positive linear operators on $C[0,1]$, the investigation of their convergence properties turns out to be much more difficult than that in the case $0 < q < 1$.

It is known that, in the case $q > 1$, the q-Bernstein polynomials approximate the entire functions and, in particular, polynomials uniformly on any compact set in $\mathbb{C}$. In this paper, the possibility of the approximation for the function $(z+a)^{\alpha}$, $a \geqslant 0$, with a non-integer $\alpha > -1$ is studied. It is proved that for $a > 0$, the function is uniformly approximated on any compact set in $\{z : |z| < a\}$, while on any Jordan arc in $\{z : |z| > a\}$, the uniform approximation is impossible. In the case $a = 0$, the results of the paper reveal the following interesting phenomenon: the power function z^{α}, $\alpha > 0$, is approximated by its q-Bernstein polynomials either on any (when $\alpha \in \mathbb{N}$) or no (when $\alpha \notin \mathbb{N}$) Jordan arc in $\mathbb{C}$.

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