

BASIC INDEXES AND ALUTHGE TRANSFORMATION FOR 2 BY 2 MATRICES

TOSHIYUKI TAKASAKI

Abstract. For 2×2 matrices we provide precise forms of their basic properties such as norms, polar decompositions and singular decompositions. Using these facts, then, we show the exact form of the Aluthge transform $\Delta(A)$ of a matrix A and give an alternative proof of Ando-Yamazaki's result of the convergence of iterations $\{\Delta^n(A)\}$. In case $|\lambda| = |\mu|$ for the eigenvalues λ, μ of A we determine the precise forms of their limits (Theorem 3).

Mathematics subject classification (2000): 15A60, 15A18.

Key words and phrases: Aluthge transform, polar decompositions, singular decompositions, singular values.

REFERENCES

- [1] A. ALUTHGE, *On p -hyponormal operators for $0 < p < 1$* , Integral Equations Operator Theory, **13** (1990), 307-315.
- [2] T. ANDO, *Aluthge transforms and the convex hull of the eigenvalues of a matrix*, Linear Multilinear Algebra, **52** (2004), 281-292.
- [3] T. ANDO AND T. YAMAZAKI, *The iterated Aluthge transforms of a 2-by-2 matrix converge*, Linear Algebra Appl., **375** (2003), 299-309.
- [4] T. YAMAZAKI, *An expression of spectral radius via Aluthge transformation*, Proc. Amer. Math. Soc., **130** (2002), 1131-1137.