

A CAUCHY–SCHWARZ INEQUALITY FOR TRIPLES OF VECTORS

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Abstract. An inequality of the Cauchy–Schwarz type is proved for triples of vectors, and interpreted geometrically in terms of the vertex angles of a tetrahedron. The results are illustrated by 3×3 correlation matrices.

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Key words and phrases: Cauchy–Schwarz inequality for three vectors; Gram matrix; Gramian; tetrahedron; angles; correlation matrices.

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