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ON THE MATRIX NORMS OF A GCD RELATED MATRIX

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Abstract. In this paper we investigate the matrix norms of a GCD related matrix, i.e., $(S_f) = \left(f(i,j)/(i^r j^r)\right)$ for multiplicative arithmetical functions f. In particular, we obtain upper bounds for the ℓ_p norms of (S_f) for $f = \varphi$, σ_α, and ψ in terms of infinite prime products. Furthermore, we give lower and upper bounds for these infinite prime products by using particular norm inequalities.

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