

L'HOSPITAL-TYPE RULES FOR MONOTONICITY: DISCRETE CASE

IOSIF PINELIS

Abstract. Let $f := (f_n)$ and $g := (g_n)$ be two sequences, and let $r := \frac{f}{g}$ and $\rho := \frac{\delta f}{\delta g}$, where $(\delta f)_n := \delta f_n := f_n - f_{n-1}$. In particular, it is shown that, if r is monotonic, then the monotonicity pattern of ρ may switch at most once (from increase to decrease or vice versa). “Continuous” versions of such results were obtained earlier by the author.

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