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A GENERAL FORM OF JORDAN'S INEQUALITIES AND ITS APPLICATIONS

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Abstract. In this work, the general form of Jordan's double inequalities:

$$P_{2N}(x)+\alpha(r^2-x^2)^{N+1}\leqslant\frac{\sin x}{x}\leqslant P_{2N}(x)+\beta(r^2-x^2)^{N+1}$$

are established, where $x\in(0,r], r\leqslant\frac{\pi}{2}, P_{2N}(x)=\sum_{n=0}^{N}a_n(r^2-x^2)^n, a_0=\frac{\sin r}{r}, a_1=\frac{\sin r-r\cos r}{2r^3}, a_{n+1}=\frac{2n+1}{2(n+1)r^2}a_n-\frac{1}{4n(n+1)r^2}a_{n-1}$, $N\geqslant 0$ is a natural number, $\alpha=a_{N+1}$ and $\beta=\frac{1-\sum_{n=0}^{N}a_nr^{2n}}{r^{2(N+1)}}$ are the best constants in inequalities above. The application of the result above give a new infinite series $(\sin x)/x=\sum_{n=0}^{\infty}a_n(r^2-x^2)^n$ for $0<|x|\leqslant r\leqslant\pi/2$, the general improvement of Yang Le inequality, and a general form of Kober's double inequality.

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