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ON SOME HIGHER–DIMENSIONAL HILBERT'S AND HARDY–HILBERT'S INTEGRAL INEQUALITIES WITH PARAMETERS

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Abstract. By introducing some parameters and a norm $|x|_\alpha$, $x \in \mathbf{R}_+^n$, we give higher-dimensional Hilbert's and Hardy-Hilbert's integral inequalities in non-conjugate case. Further, we prove that their constant factors are the best possible, in the conjugate case, when the parameters satisfy appropriate conditions. We also compare our results with some known results.

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