Mathematical
Inequalities
& Applications
Volume 11, Number 4 (2008), 739–748

EXTENSIONS OF A BONNESEN–STYLE INEQUALITY TO MINKOWSKI SPACES

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Abstract. Various definitions of surface area and volume are possible in finite dimensional normed linear spaces (= Minkowski spaces). Using a Bonnesen-style inequality, we investigate the ratio of the Holmes-Thompson surface area of the unit ball to its volume. In particular, in the planar case a stronger lower bound for this ratio is established when the area is defined in the sense of Holmes-Thompson, or is given by the definition of mass. From this we obtain some (characteristic) properties of Radon curves.

Mathematics subject classification (2000): 52A10, 52A21, 52A40, 46B20.
Key words and phrases: Benson area, Busemann area, Bonnesen inequality, convex body, Holmes-Thompson area, isoperimetrix, mass area, Minkowski plane, mixed volume, normed linear space, Petty's conjectured projection inequality, projection body, Radon curve, relative inner and outer radii, tangent body.

REFERENCES

[1] ALVAREZ, J.C., AND DURAN, C., *An Introduction to Finsler Geometry*, Notas de la Escuela Venezolana de Mathematicas, 1998.
[2] ALVAREZ, J.C. AND THOMPSON, A.C., *On the perimeter and area of the unit disc*, Amer. Math. Monthly **112** (2005), 141-154.
[3] ALVAREZ, J.C. AND THOMPSON, A.C., *Volumes in normed and Finsler spaces*, *A sampler of Riemann-Finsler geometry*, 1-48, Math. Sci. Res. Inst. Publ. **50**, Cambridge Univ. Press, Cambridge, 2004.
[4] BONNESEN, T. AND FENCHEL, W., *Theory of Convex Bodies*, BCS Associates, Moscow, Idaho USA, 1987.
[5] BRANNEN, N.S., *Volumes of projection bodies*, Mathematika **43** (1996), 255-264.
[6] DISKANT, V.I., *A generalization of Bonnesen's inequalities*, Soviet Math. Dokl. **14** (1973), 1728-1731 (transl. of Dokl. Akad. Nauk SSSR **213** (1973), no 3).
[7] DÜVELMEYER, N., *A new characterization of Radon curves via angular bisectors*, J. Geom. **80** (2004), 75-81.
[8] FAVARD, J., *Sur les corps convexes*, J. Math. Pures. Appl. (9) **12** (1933), 219-282.
[9] FLANDERS, H., *A proof of Minkowski's inequality for convex curves*, Amer. Math. Monthly **75** (1968), 581-593.
[10] GAGE, M., *Positive centers and the Bonnesen inequality*, Proc. Amer. Math. Soc. **110** (1990), 1041-1048.
[11] GRUBER, P.M., *Stability of Blaschke's characterization of ellipsoids and Radon norms*, Discrete Comput. Geom. **17** (1997), 411-427.
[12] HOLMES, R.D., AND THOMPSON, A.C., *N -dimensional area and content in Minkowski spaces*, Pacific J. Math. **85** (1979), 77-110.
[13] LAUGWITZ, D., *Konvexe Mittelpunktsbereiche und normierte Räume*, Math. Z. **61** (1954), 235-244.
[14] LUTWAK, E., *On a conjectured inequality of Petty*, Contemp. Math. 113 (1990), 171-182.
[15] MARTINI, H., AND MUSTAFAEV, Z., *Some applications of cross-section measures in Minkowski spaces*, Period. Math. Hungar. **53** (2006), 185-197.
[16] MARTINI, H. AND SWANEPOEL, K.J., *Equiframed curves – a generalization of Radon curves*, Monatsh. Math. **141** (2004), 301-314.
[17] MARTINI, H. AND SWANEPOEL, K.J., *Antinorms and Radon curves*, Aequationes Math. **71** (2006), 110-138.

[18] MARTINI, H., SWANEPOEL, K.J., WEISS, G., *The geometry of Minkowski spaces – a survey. Part I*, Expositiones Math. **19** (2001), 97-142.
[19] MARTINI, H., SWANEPOEL, K.J., WEISS, G., *The Fermat-Torricelli problem in normed planes and spaces*, J. Optim. Theory Appl. **115** (2002), 283-314.
[20] MUSTAFAEV, Z., *The ratio of the length of the unit circle to the area of the unit disk in Minkowski planes*, Proc. Amer. Math. Soc. **133** (2005), 1231-1237.
[21] OSSERMAN, R., *Bonnesen-style isoperimetric inequalities*, Amer. Math. Monthly **86** (1979), 1-29.
[22] REISNER, S., *Zonoids with minimal volume-product*, Math. Z. **192** (1986), 339–346.
[23] SANGWINE-YAGER, J.R., *Bonnesen-style inequalities for Minkowski relative geometry*, Trans. Amer. Math. Soc. **307** (1988), 373–382.
[24] SCHÄFFER, J.J., *Geometry of Spheres in Normed Spaces*, Marcel Dekker, Inc., New York-Basel, 1976.
[25] SCHNEIDER, R., *Geometric inequalities Poisson processes of convex bodies and cylinders*, Results Math. **11** (1987), 165-186.
[26] SCHNEIDER, R., *Convex Bodies: The Brunn-Minkowski Theory*, Encyclopedia of Mathematics and its Applications, Vol. **44**, Cambridge University Press, Cambridge, 1993.
[27] TAMÁSSY, L., *Über ein Problem dez zweidimensionalen Minkowskischen Geometrie*, Ann. Polon. Math. **9** (1960/61), 39-48.
[28] THOMPSON, A.C., *Minkowski Geometry*, Encyclopedia of Mathematics and its Applications, Vol. **63**, Cambridge University Press, Cambridge, 1996.