

AN INEQUALITY FOR THE TAKAGI FUNCTION

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Abstract. The well-known Takagi function $T(x) = \sum_{k=0}^{\infty} 2^{-k} \text{dist}(x, \mathbb{Z})$ plays a crucial role in the theory of approximately convex functions. In order to establish the sharpness of some Bernstein–Doetsch type results for approximate convexity, we prove that the Takagi function fulfils the inequality

$$T\left(\frac{x+y}{2}\right) \leq \frac{1}{2}(T(x) + T(y) + |x - y|)$$

for all real numbers x and y .

Mathematics subject classification (2000): 26A51, 26A30, 39B62.

Key words and phrases: Approximately Jensen-convex functions; Takagi function.

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