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A NEW SYSTEM OF GENERAL NONLINEAR VARIATIONAL INCLUSIONS INVOLVING (A,η)–ACCRETIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, a new system of general nonlinear variational inclusions involving (A,η)-accretive mappings in Banach spaces is introduced and studied, which includes many variational inequality (inclusion) problems as special cases. By using the resolvent operator technique for (A,η)-accretive mapping due to Lan-Cho-Verma, an existence and uniqueness theorem of solutions for this system of variational inclusion is proved. A new iterative algorithm for finding approximate solution of this system variational inclusion is suggested and discussed, the convergence and stability of iterative sequence generated by new iterative algorithm is also given. The theorems presented in this paper improve and unify many known results variational inequalities and variational inclusions.

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