Mathematical
Inequalities
& Applications

Volume 12, Number 1 (2009), 53–65

GENERALIZATION OF THE PEČARIĆ–RAJIĆ INEQUALITY IN NORMED LINEAR SPACES

SEVER S. DRAGOMIR

Abstract. In this paper we establish a generalization of the recent *Pečarić-Rajić inequality* by providing upper and lower bounds for the norm of the linear combination $\sum_{j=1}^{n} \alpha_j x_j$ where $\alpha_j \in \mathbb{K}$ and $x_j \in X$ for $j \in \{1, \ldots, n\}$ with $n \geqslant 2$. Applications for two vectors that are related to the *Massera-Schäffer*, *Dunkl-Williams* and *Maligranda-Mercer inequalities* are given. Some bounds for the quantity $\|x/\|y\| - y/\|x\|\|$ with $x, y \in X \setminus \{0\}$, are also provided.

Mathematics subject classification (2000): 26D15, 46B05, 26D10.
Keywords and phrases: Normed linear spaces, Triangle Inequality, Dunkl-Williams inequality, Massera-Schäffer inequality.

REFERENCES

[1] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional.* Bull. Austral. Math. Soc., **74**(3) (2006), 471–478.
[2] S. S. DRAGOMIR, *Another Grüss type inequality for sequences of vectors in normed linear spaces and applications.* J. Comput. Anal. Appl., **4**(2) (2002), 155–172.
[3] S. S. DRAGOMIR, *A Grüss type inequality for sequences of vectors in normed linear spaces and applications.* Tamsui Oxf. J. Math. Sci., **20**(2) (2004), 143–159.
[4] S. S. DRAGOMIR, *Grüss type discrete inequalities in normed linear spaces, revisited.* Nonlinear Funct. Anal. Appl., **9**(4) (2004), 577–591.
[5] S. S. DRAGOMIR, *Bounding the Cebysev functional for sequences of vectors in normed linear spaces*, Filomat, **18** (2004), 15–26.
[6] S. S. DRAGOMIR AND G. L. BOOTH, *Grüss-Lupaş type inequality and its applications for the estimation of p -moments of guessing mappings.* Math. Commun., **5**(2) (2000), 117–126.
[7] C. F. DUNKL AND K. S. WILLIAMS, *A simple norm inequality*, Amer. Math. Monthly, **71** (1964), 53–54.
[8] M. KATO, K.-S. SAITO AND T. TAMURA, *Sharp triangle inequality and its reverse in Banach spaces*, Math. Inequal. & Appl., **10**(2) (2007), 461–470.
[9] L. MALIGRANDA, *Simple norm inequalities*, Amer. Math. Monthly, **113** (2006), 256–260.
[10] J. L. MASSERA AND J. J. SCHÄFFER, *Linear differential equations and functional analysis (I)*, Ann. of Math., **67** (1958), 517–573.
[11] P. R. MERCER, *The Dunkl-Williams inequality in an inner product space*, Math. Inequal. & Appl., **10**(2) (2007), 447–450.
[12] D. S. MITRINOVIĆ, J. PEČARIĆ AND A. M. FINK, *Classical and New Inequalities in Analysis.* Mathematics and its Applications (East European Series), 61. Kluwer Academic Publishers Group, Dordrecht, 1993.
[13] J. PEČARIĆ AND R. RAJIĆ, *The Dunkl-Williams inequality with n elements in normed linear spaces*, Math. Inequal. & Appl., **10**(2) (2007), 461–470.
[14] J. PEČARIĆ, F. PROSCHAN AND Y. L. TONG, *Convex Functions, Partial Orderings, and Statistical Applications.* Mathematics in Science and Engineering, 187. Academic Press, Inc., Boston, MA, 1992.

Paper MIA-12-05

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