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UNIFORM BOUNDS FOR THE COMPLEMENTARY INCOMPLETE GAMMA FUNCTION

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Abstract. We prove upper and lower bounds for the complementary incomplete gamma function $\Gamma(a,z)$ with complex parameters a and z. Our bounds are refined within the circular hyperboloid of one sheet $\{(a,z) : |z| > c|a-1|\}$ with a real and z complex. Our results show that within the hyperboloid, $|\Gamma(a,z)|$ is of order $|z|^{a-1}e^{-\mathrm{Re}(z)}$, and extends an upper estimate of Natalini and Palumbo to complex values of z.

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