

DIFFERENTIAL INEQUALITY CONDITIONS FOR DOMINANCE BETWEEN CONTINUOUS ARCHIMEDEAN T-NORMS

SUSANNE SAMINGER-PLATZ, BERNARD DE BAETS AND HANS DE MEYER

Abstract. Dominance between triangular norms (t-norms) is a versatile relationship. For continuous Archimedean t-norms, dominance can be verified by checking one of many sufficient conditions derived from a generalization of the Mulholland inequality. These conditions pertain to various convexity properties of compositions of additive generators and their inverses. In this paper, assuming differentiability of these additive generators, we propose equivalent sufficient conditions that can be expressed as inequalities involving derivatives of the additive generators, avoiding the need of composing them. We demonstrate the powerfulness of the results by the straightforward rediscovery of dominance relationships in the Schweizer-Sklar t-norm family, as well as by unveiling some formerly unknown dominance relationships in the Sugeno-Weber t-norm family. Finally, we illustrate that the results can also be applied to members of different parametric families of t-norm.

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