

EQUALITY CONDITIONS FOR NORM INEQUALITIES IN REPRODUCING KERNEL HILBERT SPACES

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Abstract. We consider norm inequalities arising from nonlinear maps between reproducing kernel Hilbert spaces. In many cases we know that equality for such inequalities occurs only for the reproducing kernels. To investigate this phenomenon we introduce a new class of RKHSs and give fairly general equality conditions for norm inequalities.

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