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FUNDAMENTAL ITERATED CONVOLUTION INEQUALITIES IN WEIGHTED L_p SPACES AND THEIR APPLICATIONS

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Abstract. In this paper, we obtain the inequalities for the iterated convolution and their applications to physical problems. We also get the inequality

$$\left\| \sum_m \left(\prod_{j=1}^r *(F_{m,j}\rho_{m,j}) \right) \left(\prod_{j=1}^r *|\rho_{m,j}| \right)^{\frac{1}{p}-1} \right\|_{L_p(\mathbb{R}^n)} \leqslant \sum_m \prod_{j=1}^r \|F_{m,j}\|_{L_p(\mathbb{R}^n,|\rho_{m,j}|)}$$

and its applications in $L_p(\mathbb{R}^n, |\rho|)$ space.

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