CONCERNING THE INTERMEDIATE
POINT IN THE MEAN VALUE THEOREM

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Abstract. If the function $f : I \to \mathbb{R}$ is differentiable on the interval $I \subseteq \mathbb{R}$, then for each $x, a \in I$, according to the mean value theorem, there exists a number $c(x)$ belonging to the open interval determined by x and a, and there exists a real number $\theta(x) \in\,]0,1[$ such that

$$f(x) - f(a) = (x - a) f^{(1)}(c(x))$$

and

$$f(x) - f(a) = (x - a) f^{(1)}(a + (x - a)\theta(x)).$$

In this paper we shall study the differentiability of the functions c and θ in a neighbourhood of a.

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