BOUNDEDNESS OF HARDY OPERATORS
ON GENERALIZED AMALGAMS

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Abstract. Boundedness of the Hardy operator $(Tf)(x) = \int_{-\infty}^{x} f(t)dt$ between amalgam spaces $\ell^q(X_u)$ and $\ell^{\bar{q}}(L_v^{\bar{p}})$ is obtained, where X_u is a weighted Banach function space. The adjoint opertor $(T^*f)(x) = \int_x^{\infty} f(t)dt$ has also been treated.

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