Mathematical
Inequalities
& Applications
Volume 12, Number 3 (2009), 573–586

GENERAL CLOSED 4–POINT QUADRATURE FORMULAE OF EULER TYPE

I. FRANJIĆ, J. PEČARIĆ, AND I. PERIĆ

Abstract. General closed 4-point quadrature formulae where the integral over $[0,1]$ is approximated with the values of the function in points: $0, x, 1-x$ and 1 are derived. As special cases, Simpson's formula, Simpson's 3/8 formula and the Gauss 2-point formula are recaptured. Next, general corrected closed 4-point quadrature formulae are derived, and the Lobatto 4-point, corrected Simpson's, corrected Simpson's 3/8 formula and the corrected Gauss 2-point formula are obtained as special cases. We call "corrected" such quadrature formulae where the integral is approximated not only with the values of the integrand at certain points but with the values of its first derivative at the end points of the interval as well.

Mathematics subject classification (2000): 26D15, 65D30, 65D32.
Keywords and phrases: quadrature formulae, corrected quadrature formulae, Lobatto 4-point formula, Bernoulli polynomials, extended Euler formulae.

REFERENCES

[1] M. ABRAMOWITZ AND I. A. STEGUN (EDS), *Handbook of mathematical functions with formulae,graphs and mathematical tables*, National Bureau of Standards, Applied Math.Series 55, 4th printing, Washington, 1965.
[2] P.J.DAVIS, P.RABINOWITZ, *Methods of Numerical Integration*, New York, 1975.
[3] LJ.DEDIĆ, M.MATIĆ, J.PEČARIĆ, *On generalizations of Ostrowski inequality via some Euler-type identities*, Math. Inequal. Appl., **3**, 3 (2000), 337–353.
[4] LJ. DEDIĆ, M. MATIĆ, J. PEČARIĆ AND A.VUKELIĆ, *On Euler-Simpson 3/8 formulae*, to appear in Nonlinear Studies.
[5] LJ. DEDIĆ, M. MATIĆ, J. PEČARIĆ, *On Euler-Simpson formulae*, PanAmer.Math.J., **11**, 2 (2001), 47–64.
[6] I.FRANJIĆ, J.PEČARIĆ, I.PERIĆ, *Quadrature formulae of Gauss type based on Euler identities*, Math. Comput. Modelling, **45**, 3–4 (2007), 355–370.
[7] I.FRANJIĆ, J.PEČARIĆ, *On corrected Euler-Simpson's 3/8 formulae*, Nonlinear Stud., **13**, 4 (2006), 309–319.
[8] I.FRANJIĆ, J.PEČARIĆ, *Generalisation of corrected Simpson's formula*, ANZIAM J., **47** (2006), 367–385.
[9] F.B.HILDEBRAND, *Introduction to Numerical Analysis*, McGraw-Hill Book Company Inc., New York, 1956.
[10] V. I. KRYLOV, *Approximate calculation of integrals*, Macmillan, New York-London, 1962.
[11] C.LANCZOS, *Applied Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1956.
[12] J.PEČARIĆ, I.PERIĆ, A.VUKELIĆ, *Sharp integral inequalities based on general Euler two-point formulae*, ANZIAM J., **46** (2005), 555–574.
[13] N.UJEVIĆ AND A.J.ROBERTS, *A corrected quadrature formula and applications*, ANZIAM J. 45 (E) (2004), pp. E41–E56.

Paper MIA-12-45