ITERATIVE APPROXIMATIONS FOR MULTIVALUED
NONEXPANSIVE MAPPINGS IN REFLEXIVE BANACH SPACES

YISHENG SONG AND YEOL JE CHO

Abstract. In this paper, we established the strong convergence of Browder type iteration $\{x_t\}$ for the multivalued nonexpansive nonself-mapping T satisfying the weakly inwardness condition in a reflexive and strictly convex Banach space E with a uniformly Gâteaux differentiable norm or in a reflexive Banach space with weakly sequentially continuous duality mapping. Furthermore, we also obtained the strong convergent results for the Halpern type iteration $\{x_n\}$ for multivalued nonexpansive nonself-mapping T.

Mathematics subject classification (2000): 47H05, 47H10, 47H17.

Keywords and phrases: Multivalued nonexpansive mapping, strong convergence, reflexive and strictly convex Banach space, weakly sequentially continuous duality mapping, uniformly Gáteaux differentiable norm..

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Paper MIA-12-47

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