

A BEST POSSIBLE INEQUALITY FOR CURVATURE-LIKE TENSOR FIELDS

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Abstract. We give an inequality for curvature-like tensor fields and apply this to Lagrangian submanifolds of complex space forms and to centroaffine hypersurfaces. In both settings we investigate the equality case and give a classification theorem if equality is attained at every point of the submanifold. We also provide an example showing that this inequality is best possible, in a sense explained in the paper.

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