

CONDITIONS CHARACTERIZING THE HARDY AND REVERSE HARDY INEQUALITIES

A. KUFNER, K. KULIEV, AND G. KULIEVA

Abstract. We summarize the conditions characterizing the Hardy and reverse Hardy inequalities for the case $p, q \in \mathbb{R} \setminus \{0\}$, and extend in some cases the number of equivalent conditions.

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